

**UNITED STATES DISTRICT COURT  
FOR THE NORTHERN DISTRICT OF ALABAMA**

EVAN MILLIGAN, et al.,

*Plaintiffs,*

v.

JOHN H. MERRILL, et al.,

*Defendants.*

Civil Case No. 2:21-CV-01530-AMM

**PLAINTIFFS' THIRD EVIDENTIARY SUBMISSION**

In further support of their motion for a preliminary injunction and following the production of Exhibits 1 to 5 in Plaintiffs' First Evidentiary Submission (Doc. 68) and Exhibits 6 to 14 in Plaintiffs' Second Evidentiary Submission (Doc. 70), Plaintiffs hereby submit the following additional Exhibits as set out below:

- Exhibit 15 – Rebuttal Expert Report of Baodong Liu, Ph.D.
- Exhibit 16 – Supplemental Declaration of Joseph Bagley Ph.D.
- Exhibit 17 – Rebuttal Expert Report of Kosuke Imai, Ph.D.
- Exhibit 18 – Rebuttal Expert Report of Moon Duchin, Ph.D.

DATED this 21st day of December 2021.

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## CERTIFICATE OF SERVICE

I hereby certify that on December 21, 2021, a true and correct copy of the foregoing was served on all counsel of record by electronic mail.

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**IN THE UNITED STATES DISTRICT COURT  
NORTHERN DISTRICT OF ALABAMA  
SOUTHERN DIVISION**

EVAN MILLIGAN, et al.,

*Plaintiffs,*

vs.

JOHN H. MERRILL, et al.,

*Defendants.*

No. 2:21-cv-01530-AMM

**Rebuttal Report of Baodong Liu, Ph.D.**

**December 20, 2021**

I have been asked to express my opinion on the expert report of Dr. M.V. Hood III, an expert for the Defendants in the above captioned litigation. This report serves as a rebuttal to Dr. Hood's report dated December 10, 2021.

This rebuttal summarizes the areas of agreement and the limitations of Dr. Hood's report.

**Areas of Agreement with Dr. Hood**

Despite my concerns about his methodology, Dr. Hood and I agree in four important areas. First, Dr. Hood rightly concluded in his report that "racially polarized voting is present [in Alabama] with black voters overwhelmingly supporting the Democratic candidate and more than a majority of white voters casting a ballot for the Republican candidate." (Hood p. 13) Second, Dr. Hood and I agree that white bloc voting will usually result in the defeat of Black-preferred candidates in white-majority districts in Alabama. (Hood p. 14, Liu p. 18) Third, Dr. Hood is correct about the necessity of using Gary King's ecological inference (ei) method for estimating the candidate of choice for different racial groups (Hood p. 3) Finally, Dr. Hood and I agree that, "[i]n a Democratic primary, white and black voters may support different candidates. If there is an insufficient number of black voters to constitute a majority in a Democratic primary, the black community may be unable to elect their candidate of choice." (Hood p. 14). Indeed, my initial report used ei to show racially polarized voting in the 2020 congressional district (CD) 1 primary election (Liu p. 10) and I reviewed exit poll data that showed racially polarized voting in the 2008 Democratic presidential primaries (Liu p. 14).

### **The Methodology in Dr. Hood's Study of Racial Turnout Rates**

Dr. Hood rightly acknowledged the need to consider racial turnout disparities when offering his opinion on functionality analysis (FA). His method for his FA was an attempt to predict what will happen in the future given how different plans including the "enacted plan" provide different opportunity structures for racial groups to vote for their candidate of choice.

In his first step of FAs, Dr. Hood used *ei* to derive his racial group vote estimates for candidates. For example, his Table 1 shows the racial estimates (Black, White and Other) for the vote choice between the Democratic candidate (Biden) and the Republican candidate (Trump) by using the 2020 Presidential election dataset. His Table 1 indicates the racially polarized voting (RPV) results between Black and white voters with respect to the enacted CD 7.

Strangely, after showing the results of RPV in Table 1, Dr. Hood went on to estimate racial turnout disparities by using what he labeled as "historical registration and turnout data". This procedure is odd because his Table 1 results were already derived along with the racial turnout disparities. To be more specific, the *ei* package he used (*eiPack*) and the *RxC* procedure in his *ei* operation allowed him to estimate racial turnouts as well as racial vote estimates for candidates. This is the appropriate approach for his FA, and his R-code in his "Replication" folder showed that he indeed engaged in such R operation. Thus, he should already have had his racial turnout rates as he completed his Table 1 procedure. But Dr. Hood choose not to report these racial turnout estimates from his own *ei* operations. Instead, he went further to use a different dataset and a different method to derive his Tables 2 and 3 about racial turnout breakdown in enacted CD 7.

After being asked to provide a detailed explanation for how his method for arriving at the results reported in his Tables 2 and 3, Dr. Hood did not provide the requested explanation.

### **The Selected Elections in Dr. Hood's Report**

In Dr. Hood's published article, "From Legal Theory to Practical Application: A How-To for Performing Vote Dilution Analyses," the appropriate approach to an RPV analysis, according to Dr. Hood and his two co-authors, "must also consider the race/ethnicity of the candidates running for election. Of the elections available for analysis, the more relevant are those that feature a minority candidate from the racial/ethnic group suing the jurisdiction in question. For example, in a vote dilution suit brought by Latino voters, one would seek election contests featuring Hispanic candidates, while also keeping in mind the other criteria previously discussed" (Hood, Morrison and Bryan, 2017, p.546).<sup>1</sup> But the two elections Dr. Hood analyzed (i.e., the 2020 Presidential Election and the 2018 Gubernatorial Election) did not directly involve a minority candidate. The 2018 Gubernatorial Election did not involve a minority candidate at all. Though the Democratic Vice-Presidential candidate was a minority (Black/Asian) candidate

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<sup>1</sup> M.V. Hood III, Peter A. Morrison, and Thomas M. Bryan. 2017. "From Legal Theory to Practical Application: A How-To for Performing Vote Dilution Analyses." *Social Science Quarterly* 99 (2): 536-552.

(Kamala Harris), the 2020 Presidential Election featured two white men on the top of the tickets for both major parties.<sup>2</sup>

### **The Misleading Assertion about Black Republican Candidate “Success” in Dr. Hood’s Report**

Dr. Hood next switched his attention to “minority Republican candidates” (p. 15). Arguing that “white conservatives support minority Republican candidates at the same rates or at significantly higher rates than Anglo (non-Hispanic white) GOP nominees”, Dr. Hood attempted to relate what happened in Alabama to his own 2015 publication on Public Opinion Quarterly. Without doing any RPV analysis for a single election that did take place in Alabama, Dr. Hood cited Kenneth Paschal from HD 73 as an example for his claim. Paschal won the Republican runoff election in 2021 with 51.1% votes cast, according to Dr. Hood, and he defeated his white Democratic opponent in the Special General Election at the end with 74.7% of the vote.

But as Dr. Hood indicated, Paschal, as an African American, “ran in a Shelby County district which is 84.1% white VAP.” Such a super white-majority district, unfortunately, does not allow any realistic opportunity to estimate the extent to which RPV, or lack thereof, may have any influence on the election outcome in a typical racially contested election in Alabama. To verify Dr. Hood’s claim, I ran an RxC ei operation by using the precinct-level election data from the 2021 special election in HD 73. The results of my RPV analysis shows that it is indeed an unreliable election to estimate white support for a Black Republican candidate. The turnout was low overall at 5.3% of the voting-age population. Especially among the white electorate, only 1.7% of the white voting-age population turned out to vote, which suggests that white voters were not highly interested in this election featuring a Black Republican candidate. Furthermore, both white and black racial vote estimates had an extremely large confidence interval<sup>3</sup> to the extent that the wide range for the ei results are not useful and cannot be taken seriously. The white vote, for instance, may be as low as 22% for Paschal or as high as 88.9%, while his Black support was similarly estimated between 15% and 72%.

To gauge the willingness of white voters in Alabama to vote for a Black Republican candidate, one should pay attention to state-wide elections where white voters are given a chance to vote for a Black Republican candidate with high name-recognition in a racially contested election.<sup>4</sup> To evaluate Dr. Hood’s claim, I conducted a RPV analysis of the 2016 Republican Presidential Primary in Alabama in which Ben Carson, a highly publicized Black candidate, ran against ten other candidates including President Donald Trump.

I report the RPV findings about this election in Table A below. Ben Carson, as shown in the table, received only about 9% of the white vote in Alabama. In contrast, Carson received about 31% of the Black Republican vote. Thus, Black Republicans were over three times more likely

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<sup>2</sup> As a verification study, I ran a RxC ei operation for the 2020 Presidential election, and the state-wide results showed that indeed it was highly racially polarized in that Biden/Harris won around 95% of the Black vote and only 12% of the white vote.

<sup>3</sup> I explained confidence intervals in footnote 10 of my initial report.

<sup>4</sup> For example, national polls from October 2015 showed Carson as the lead Republican candidate. NBC/WSJ Poll: Carson Surges Into Lead of National GOP Race (Nov. 2, 2015), <https://www.nbcnews.com/politics/2016-election/nbc-wsj-poll-carson-surges-lead-national-gop-race-n456006>.

than whites to support Carson. Donald Trump, on the other hand, received more than 44% of the white vote and essentially tied with Carson with 33% of the Black Republican vote. When the primary outcome was announced, Trump was the overwhelming winner with more than 43% of the total votes cast while Carson was in the fourth place with barely over 10% of the votes.


Table A: RPV in the 2016 Republican Presidential Primary, Alabama

<b>Group</b>	<b>Turnout</b>	<b>Carson</b>	<b>Trump</b>	<b>All-others</b>
Black	0.013 (0.011, 0.022)	0.307 (0.268, 0.338)	0.333 (0.299, 0.368)	0.36 (0.326, 0.397)
White	0.312 (0.296, 0.319)	0.089 (0.078, 0.094)	0.447 (0.443, 0.455)	0.464 (0.461, 0.467)
Total	0.217	0.103	0.439	0.458

It is also worth noting that only 1.3% of Black voters participated in this Republican primary. Dr. Hood's assertion of the white conservative support for Black Republican candidates in Alabama has little, if any, empirical support.

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Per 28 U.S. Code § 1746, I declare under penalty of perjury that the forgoing is true and correct. Executed on December 20, 2021.

  
Baodong Liu, Ph.D.



**MILLIGAN V. MERRILL**  
**Case No.: 2:21-cv-012921**  
**SUPPLEMENTARY DECLARATION OF JOSEPH BAGLEY, PHD**  
**REBUTTAL OF REPORT OF THOMAS M. BRYAN**

Thomas M. Bryan asserts in his report for the defendants that Mobile and Baldwin Counties constitute an inseparable community of interest (“COI”) and that splitting these counties, as in the *Milligan* plaintiffs’ proposed plan, would “cause the most harm” among county splits in said plan. Mr. Bryan also alludes to the Black Belt region of the state but does not explain the historical, demographic, or socioeconomic characteristics of the region. In my opinion, the Bryan report fails to describe the Black community and the Black Belt and its close relationship to the Black people of Mobile.

The Black Belt is a region that stretches across America’s Deep South, from South Carolina to Texas. It is named for its rich black soil. Though the majority of the American Black Belt’s inhabitants are also Black people, the descendants of the enslaved who were forced to work that land before and during the Civil War.

The Alabama Black Belt extends, roughly, from Russell and Barbour Counties in East Alabama, through Montgomery County, to an expanding area covering Pickens County to Washington County on the Mississippi line.

As Native Americans were gradually and forcibly removed from the lands west of the Ocmulgee River in the late 18<sup>th</sup> and early 19<sup>th</sup> centuries, white settlers realized that the Black Belt’s soil, and the Deep South’s climate, were perfect for growing long-staple cotton. At the same time, the invention of the cotton gin and the beginnings of industrialization increased demand for that crop, and a decline in the tobacco market created a “surplus” of enslaved Black people in the older plantation areas of the Tidewater of Virginia and North Carolina.

White settlers began to flood into the state of Alabama when most of the remaining Creek Indians were forced out via the Indian Removal Act of 1830. By then, the United States government had banned the importation of slaves from abroad, so many settlers brought enslaved Black people with them from the older plantation areas of the Upper South. Others purchased them from slave markets in Montgomery, Mobile, Jackson, and other cities. American chattel slavery expanded dramatically between that time and the Civil War, giving rise to the “Cotton Kingdom” of the antebellum era when cotton was America’s most valuable export and enslaved Black people were its most valuable commodity. The Black Belt of Alabama became home to not only the wealthiest white plantation owners in the state, but to some of the wealthiest individuals in the young nation, some of whom held hundreds of people in bondage.

When the 13<sup>th</sup> Amendment brought an end to chattel slavery, land was never systematically redistributed from white landowners and given to newly freed Black people. Formerly enslaved Black people became landless tenant farmers, beholden to their former

masters. And when Alabama replaced its constitution in 1875 and again in 1901, it was the “Bourbon redeemers” of the Black Belt region, hyper-wealthy white landowners, who pushed hardest for a document that would protect white supremacy. Black people were the overwhelming majority in most areas. The Black Belt’s white landowners feared that allowing Black people to vote freely would lead to land reform and their political and financial ruin. Thus, they lobbied for protections against white property tax dollars for Black education and for the total disenfranchisement of Black citizens.

When the nonviolent movement for civil rights reached its peak in the mid-1950s, it was the Black Belt where Black activists faced the most formidable reprisals – violent and economic. The Black Belt was also the seedbed of both the Ku Klux Klan and the Citizens’ Council in the state. The Citizens’ Councils ensured that any Black people engaged in civil rights activism received “the pressure,” meaning they would be fired by white employers, evicted by white landowners, denied credit by white bankers, etc.<sup>1</sup> “Bloody Sunday” occurred in the Black Belt city of Selma, and the related murder of Viola Liuzzo occurred in nearby Lowndes County, dubbed “Bloody Lowndes” for the violence meted out against voting rights protestors.<sup>2</sup> White people fled public schools in the Black Belt rather than integrate and even fled some cities entirely rather than share local governmental power.<sup>3</sup>

The Black Belt was also the site of Black citizens’ efforts to organize and to seek access to the franchise and to equal educational opportunity. When the National Association for the Advancement of Colored People encouraged local branches to petition school boards to address the Supreme Court’s *Brown v. Board of Education* decision in 1955, Black people in Butler, Russell, Bullock, and Dallas Counties were among those to answer the call (Black activists in Mobile did the same). The Lowndes County Freedom Association was founded in 1965 and the National Democratic Party of Alabama was formed soon thereafter with both independent focused on running Black candidates in elections in the Black Belt.<sup>4</sup>

White backlash to Black activism took the form of violence and economic reprisals, which contributed to Black Alabamians’ migration from the Black Belt to Mobile and elsewhere as early as the end of the Civil War. This migration of Black people from the Black Belt to Mobile continued through the end of the Nineteenth Century and into the Twentieth Century.

The historian Wayne Flynt has described a “massive hemorrhaging of people,” mostly Black people, from the Black Belt, in the early Twentieth Century. As Flynt explains, “These internal migrants generally headed for cities.” This would include Black people who left the Black Belt for Mobile in significant numbers during the Great Depression, when white

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<sup>1</sup> Joseph Bagley, *The Politics of White Rights: Race, Justice, and Integrating Alabama’s Schools* (Athens: University of Georgia Press, 2018).

<sup>2</sup> Hasan Kwame Jeffries, *Bloody Lowndes: Civil Rights and Black Power in Alabama’s Black Belt* (New York: New York University Press, 2010); James P. Turner, *Selma and the Liuzzo Murder Trials: The First Modern Civil Rights Convictions* (Ann Arbor: University of Michigan Press, 2018).

<sup>3</sup> Bagley, *The Politics of White Rights*.

<sup>4</sup> Bagley, *The Politics of White Rights*; Jeffries, *Bloody Lowndes*.

landowners refused to pass down federal aid to their sharecropping tenant farmers. In the second half of the Twentieth Century, consolidation of land, mechanization, and the rise of the Sunbelt generated, in Flynt's words, "a hemorrhaging of people [from the Black Belt] even more severe" than the previous one. Again, Black people left the Black Belt for Mobile. By the end of the century, more Black people in Alabama lived in cities than in rural areas. Many Black families in Mobile are Black Belt migrants or the descendants thereof.<sup>5</sup>

As the political scientist Richard Pride writes of Mobile, "Its roots followed the rivers north into the heart of the black belt . . . where cotton and timber grew abundantly, and planters, rednecks, and blacks marked all the society that people acknowledged." Pride continues, "The city had its face turned toward the world, but it nevertheless grew out of the Old South."<sup>6</sup>

White flight accelerated significantly in Mobile when the city's long-running school desegregation case finally yielded positive results for Black plaintiffs in the early 1970s, at the same time that Black Belt public school systems were experiencing similar backlash and flight.<sup>7</sup> As in the Black Belt, white flight has left most public schools east of I-65 in Mobile overwhelmingly Black. The Black communities of Mobile and the Black Belt share significant historic, demographic, and socioeconomic interests.

I am aware that the State Board of Education ("SBOE") elects eight-members from single-member districts, including two majority Black districts. I am also aware that the parties in this case have agreed that, "[i]n each election since 2011, a Black Democrat won a majority of Black voters and the election in Districts 4 and 5 of the SBOE" and that "District 5 of the SBOE Plan connects the City of Mobile to the Black Belt Counties."<sup>8</sup> The fact that most Black voters in SBOE District 5 vote for the same candidates and the State Legislature's decision to place the Black communities in the City of Mobile and the Black Belt in the same SBOE district are consistent with my conclusions here.

In his analysis of Mobile and Baldwin Counties, Mr. Bryan relies exclusively upon the previous testimony of U.S. Congressional Representative Bradley Byrne and former Representative Jo Bonner, two white men elected from the overwhelmingly white 1<sup>st</sup> District who have asserted that Mobile and Baldwin form a sensible COI. But the population of the Mobile County east of Interstate 65 is overwhelming Black and shares little today with the rest of the metropolitan area, which is predominately white. And to the extent that western Baldwin County shares economic interest with the city, it is because safely white communities

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<sup>5</sup> Wayne Flynt, *Alabama in the Twentieth Century* (Tuscaloosa: University of Alabama Press, 2004), pp. 115, 143, 177.

<sup>6</sup> Richard Pride, *The Political Use of Racial Narratives: School Desegregation in Mobile, Alabama, 1954-1997* (Champaign: University of Illinois Press, 2002); Scotty E. Kirkland, "Pink Sheets and Black Ballots: Politics and Civil Rights in Mobile, Alabama, 1945-1985," M.A. Thesis (University of South Alabama, 2009).

<sup>7</sup> *Davis v. Mobile Board of School Commissioners*, 430 F.2d 883, 889 (5th CCA, 1970), reversed, 402 U.S. 33 (1971).

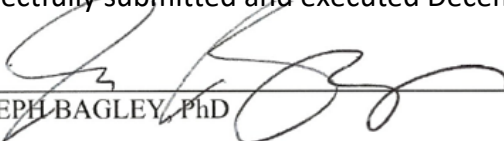
<sup>8</sup> Joint Stipulated Facts for Preliminary Injunction Proceedings, *Milligan v. Merrill*, Dec. 7, 2021.

like Fairhope, Spanish Fort, and Daphne became white flight destinations when courts called for compulsory school desegregation and white residents fled from the possibility of their kids attending majority Black Williamson High and Vigor High or a substantially Black Murphy High.<sup>9</sup> The remaining areas of Baldwin County are either sparsely populated or are Gulf Coast beach tourist destinations that have little meaningful connection to the city of Mobile save for waterfront access.<sup>10</sup>

In conclusion, it is my opinion that the Black communities in the Black Belt and Mobile County have longstanding, organic, and meaningful connections.

Pursuant to 28 U.S.C. § 1746, I declare under penalty of perjury that the forgoing is true and correct to the best of my knowledge.

Respectfully submitted and executed December 20, 2021.

  
JOSEPH BAGLEY, PhD

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<sup>9</sup> Bagley, *The Politics of White Rights*; Brian Duke, "The Strange Career of Birdie Mae Davis: A History of a School Desegregation Lawsuit in Mobile, Alabama, 1963 – 1997," M.A. Thesis, Auburn University (2009).

<sup>10</sup> Allen Tullos, *Alabama Getaway: The Political Imaginary and the Heart of Dixie* (Athens: University of Georgia Press, 2011); Harvey Jackson, *The Rise and Decline of the Redneck Riviera: An Insider's History of the Florida-Alabama Coast* (Athens: University of Georgia Press, 2013).

IN THE UNITED STATES DISTRICT COURT NORTHERN DISTRICT OF  
ALABAMA SOUTHERN DIVISION

Milligan *et al.*

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**REBUTTAL EXPERT REPORT**

**Kosuke Imai, Ph.D.**

**December 20, 2021**

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**REBUTTAL EXPERT REPORT**

**I. INTRODUCTION AND SCOPE OF WORK**

1. My name is Kosuke Imai, Ph.D., and I am a Professor in the Department of Government and the Department of Statistics at Harvard University. I specialize in the development of statistical methods for and their applications to social science research. I am also affiliated with Harvard's Institute for Quantitative Social Science. My qualifications and compensation are described in my initial report.

2. I understand from Plaintiffs' counsel that one of Defendants' experts offered the opinion that Mobile and Baldwin Counties are communities of interest and should not be divided across congressional districts. I also understand from Plaintiffs' counsel that there is evidence supporting the Black Belt, as defined below, as a community of interest. I express no opinions on these issues.

3. I have been asked by Plaintiff's counsel to re-run my "one-MMD (majority-minority district) simulation" from my initial report with additional weighting that encourages the algorithm to keep Mobile/Baldwin and the Black Belt together and to examine the likely effect on the range of black voting-age population (BVAP) proportion of non-MMD districts, particularly District 2. The original one-MMD simulation I conducted for my initial report generated 10,000 alternative plans, each of which was designed to have exactly one MMD with the proportion of black voting-age population (BVAP) ranging from 50% to 51%. The other six districts of each simulated plan were generated without any consideration of race. This time, however, I instructed the algorithm to generate, with a high probability, plans which keep Mobile and Baldwin Counties together and the Black Belt together. Other than this additional weight, the new one-MMD simulation procedure I employed is identical to the one used in my initial expert report. Like the original one-MMD simulation, my new 10,000 simulated plans are, on average, more compact and have no more county splits than the enacted plan.

**REBUTTAL EXPERT REPORT**

**II. SUMMARY OF OPINIONS**

4. The comparison of the new one-MMD simulated plans with the enacted plan yields the following findings: the district with the second highest BVAP proportion in simulated plans achieves, on average, 6.2 percentage points higher BVAP proportion than the enacted plan. This difference is statistically significant using the conventional standard. The new one-MMD simulations generated many more plans with a greater BVAP proportion for the second highest BVAP district than my initial one-MMD simulation, which did not encourage the algorithm to avoid splitting Mobile/Baldwin Counties and the Black Belt.

5. My simulation analyses, therefore, provide evidence that race was a significant factor in drawing the enacted plan, and that, taking into account the identified communities of interest, the enacted plan is still an outlier in terms of how it cracks the Black community.

**III. METHODOLOGY**

6. The simulation procedure used for this report is identical to that of the one-MMD simulation from my initial report with the exception of one additional weighting I added to discourage the simulation algorithm from splitting Mobile and Baldwin Counties as well as the Black Belt. I was instructed by Plaintiffs' counsel to use the following set of counties for the Black Belt: Barbour, Bullock, Butler, Choctaw, Clarke, Conecuh, Crenshaw, Dallas, Escambia, Greene, Hale, Lowndes, Macon, Marengo, Monroe, Montgomery, Perry, Pickens, Pike, Russell, Sumter, Washington, and Wilcox Counties. As standard in the literature, I used the so-called Gibbs measure to incorporate this constraint into the simulation algorithm (Autry et al. 2020; Carter et al. 2019; McCartan and Imai 2020; Kenny et al. 2021).<sup>1</sup> One MMD whose BVAP proportion is between 50% and 51% was generated for each simulated plan in the exactly same manner as done in the one-MMD simulation for my initial report. Finally, I used the same data set as the one analyzed in my initial report.

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1. Specifically, I used the indicator variable for splitting each of these two county clusters with a penalty weight of 25.



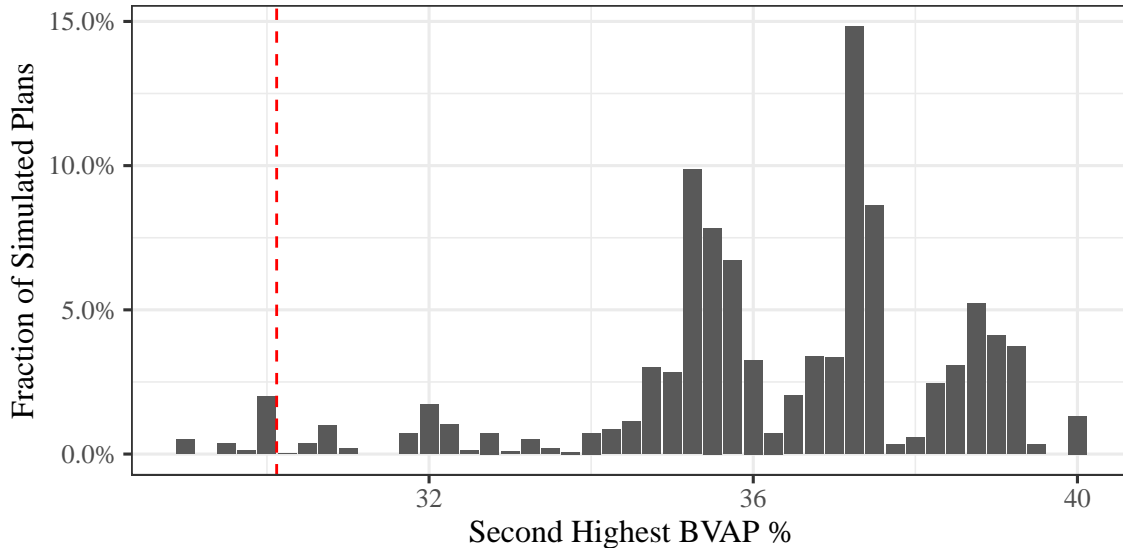
**REBUTTAL EXPERT REPORT**

Figure 1: The second highest Black voting age population (BVAP) proportion (after the simulated majority-minority district) in each simulated plan. The vast majority of simulated plans have greater BVAP than the enacted (red).

#### **IV. EVALUATION OF THE ENACTED PLAN**

7. Using the redistricting simulation methodology described above, I evaluate evidence regarding whether race was a primary factor in drawing the enacted plan. In Appendix A, I demonstrate that the simulated plans are on average at least as compact as the enacted plan based on the standard compactness measures. Appendix B further shows that all of the simulated plans have fewer than or equal to the number of county splits the enacted plan does. In addition, like the original one-MMD analysis conducted for my initial report, all simulated plans have at most one incumbent located in any given district.

8. I can easily generate additional plans by running the algorithm longer, but for the purpose of my analysis, 10,000 simulated plans for each set will yield statistically precise conclusions. In other words, generating more than 10,000 plans, while possible, will not materially affect the conclusions of my analysis.

9. Figure 1 shows the distribution of BVAP proportion for the district that has the second highest BVAP proportion under each simulated plan. Note that under more than 99% of the simulated plans, District 2 has the second highest BVAP proportion. When compared to the en-

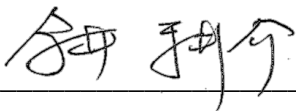
**REBUTTAL EXPERT REPORT**

acted plan (represented by the red dashed line), under the simulated plans, this district has a much higher BVAP proportion with a maximum value of 39.9%. Although all of non-MMD districts were generated without using any information about race, the simulation plan has, on average, the second highest district-level BVAP proportion at 36.3%, which is 6.2 percentage points higher than the corresponding BVAP proportion under the enacted plan (30.1%). Only 3% of the simulated plans have the second highest district-level BVAP proportion to be less than the one for the enacted plan. In other words, this difference between the simulated plans and the enacted plan is statistically significant.

10. When compared to the original one-MMD simulation reported in my initial report, this new one-MMD simulation generated many more plans with a greater BVAP proportion for the second highest district-level BVAP proportion. This implies that keeping Mobile and Baldwin Counties together and the Black Belt together is likely to significantly increase the second highest district-level BVAP proportion.

Pursuant to 28 U.S.C. § 1746, I hereby declare under penalty of perjury that the forgoing is true and correct:

Executed, this day, December 20, 2021, in Cambridge, Massachusetts.



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Kosuke Imai, Ph.D.

**REBUTTAL EXPERT REPORT**

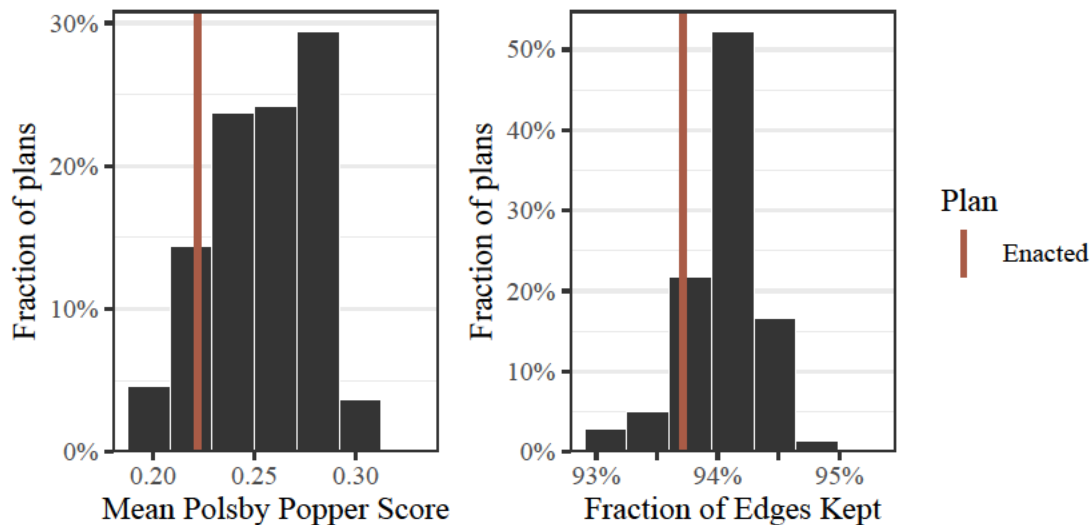


Figure 2: The compactness of the one-MMD simulated plans according to two measures – Polsby-Popper compactness (left) and fraction of edges kept (right). In general, simulated plans are as compact or more compact than the enacted plan.

## V. APPENDIX

### A. Compactness of the Simulated Plans

1. I now show that the simulated plans are more compliant than the enacted plan. I use the average Polsby–Popper (Polsby and Popper 1991) and edge-removal (DeFord, Duchin, and Solomon 2021; McCartan and Imai 2020) scores, two commonly-used quantitative measures of district compactness. Figure 2 also shows that according to the Polsby–Popper and edge-removal scores, the new one-MMD simulated plans are, on average, more compact than the enacted plan.

### B. County Splits of the Simulated Plans

2. Figure 3 presents the number of counties split within the MMD (left plot) and the total number of counties split (right plot). The figure shows that when compared to the enacted plan, all of the new one-MMD simulated plans have fewer or equal number of county splits within the MMD and across all districts.

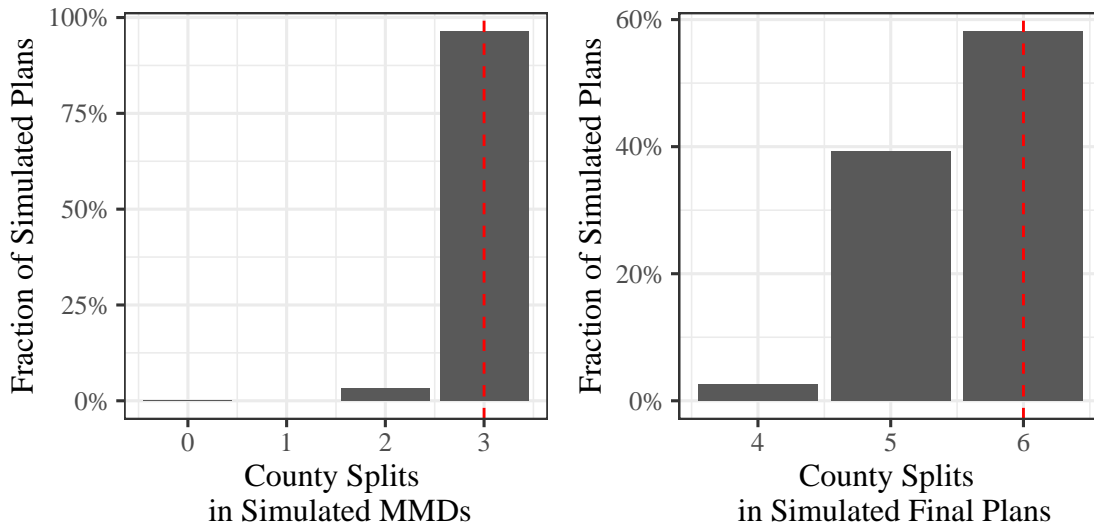
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Figure 3: The number of county splits in each simulated majority-minority district (left) and in the complete simulated plans (right). All simulated plans used in the analysis have the same number or fewer splits than the enacted plan (red).

**C. References**

- Autry, Eric, Daniel Carter, Gregory Herschlag, Zach Hunter, and Jonathan Mattingly. 2020. “Multi-scale merge-split Markov chain Monte Carlo for Redistricting.” *arXiv preprint arXiv:2008.08054*.
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- Kenny, Christopher T., Shiro Kuriwaki, Cory McCartan, Evan Rosenman, Tyler Simko, and Kosuke Imai. 2021. “The Use of Differential Privacy for Census Data and its Impact on Redistricting: The Case of the 2020 U.S. Census.” *Science Advances* 7, no. 41 (October): 1–17.

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McCartan, Cory, and Kosuke Imai. 2020. “Sequential Monte Carlo for sampling balanced and compact redistricting plans.” *arXiv preprint arXiv:2008.06131*.

Polsby, Daniel D, and Robert D Popper. 1991. “The third criterion: Compactness as a procedural safeguard against partisan gerrymandering.” *Yale Law & Policy Review* 9 (2): 301–353.

# Response to Report of Thomas Bryan

Moon Duchin  
Professor of Mathematics, Tufts University  
Collaborating Faculty in Race, Colonialism, and Diaspora Studies  
Senior Fellow, Tisch College of Civic Life

December 20, 2021

## 1 Background and assignment

I am a Professor of Mathematics and a Senior Fellow in the Jonathan M. Tisch College of Civic Life at Tufts University. I have previously submitted an expert report in the current case and have been asked by counsel to provide additional material providing my opinion on the report of Thomas Bryan, particularly focused on his discussion of compactness metrics and of racial categories on the Census.

## 2 Compactness metrics

Part 4(D) of the Thomas Bryan report (pages 29-30) covers the topic of compactness metrics. In that Part, four compactness metrics are presented: Polsby-Popper, Schwartzberg, Reock, and Convex Hull.

### 2.1 Erroneous calculation

The Schwartzberg scores are calculated incorrectly in Mr. Bryan's report. Quoting the original 1966 paper where the score was proposed by Joseph Schwartzberg<sup>1</sup>

For any given two dimensional area the most compact shape is a circle. No other geometric figure has as low a ratio between its perimeter and area. The relative compactness of any other figure may be determined by finding the ratio of its perimeter to the perimeter of a circle of equal area. The ratio serves as an index of compactness. The index number of a circle is taken to be one. All other indices are higher and represent varying degrees of departure from perfect compactness. Thus, the index number of a perfect square is 1.13, of an equilateral triangle 1.29, and of a perfect five point star 1.95.

As this makes clear, the Schwartzberg score takes a minimum value of 1 (realized only for perfect circles); all other shapes have values above that. In the Thomas Bryan report, all districts are reported to have Schwartzberg scores less than one. Mr. Bryan supports his calculation by citing the website ([fisherzachary.github.io/public/r-output.html](https://fisherzachary.github.io/public/r-output.html)) of an undergraduate student project, and including screenshots from that project in his report.

<sup>1</sup>Joseph E. Schwartzberg, Reapportionment, Gerrymanders, and the Notion of Compactness, 50 Minn. L. Rev. 443, 452 (1966).

## 2.2 Questionable combination

In addition to reporting scores incorrectly, Mr. Bryan also performs an operation that violates best practices in statistics and mathematical modeling: he adds scores that are in different units to create a "Total." Polsby-Popper scores are in dimensionless units that can be interpreted as a proportion of a certain circle's area; Reock scores are in proportion of a different circle's area; Convex Hull scores are in percentage of a certain polygon's area. It is unclear how one might interpret their sum, as the standard practice in quantitative analysis would be to only compute sums and averages of scores in like units.

The practice of summarizing multiple compactness scores of numerous districts in a single number is not just abstractly discouraged, but has a concrete impact: it serves to hide the fact that different compactness scores reward or penalize different kinds of features. This can mislead readers into thinking that two plans are directly comparable when in fact one is stronger in some ways while the other is stronger in other ways. In a case like this, the appropriate conclusion would be that the compactness comparison is marked by tradeoffs.

## 3 Racial population categories

Part 3 of the Thomas Bryan report (pages 9-13) discusses Census Race Definitions, tallying population with categories that he calls "Black Alone" and "All Black." Mr. Bryan writes on p.10 that "the "alone" definition has been most defensible from a political science / Gingles 2 voting behavior perspective"—here, it is unclear what references support his claim, from political science or any other scholarly or practitioner literature.

As Mr. Bryan acknowledges, the ability to use multiple categories to self-identify race in the Census is relatively recent. I note the Decennial Census treats Black as a checkbox, i.e., a yes/no question (see Figure 1). Thus, the Any-Part-Black definition (AKA "All Black") can be very simply described: *it contains all residents who, when presented with the Yes-or-No question about whether they are Black, answered Yes.*

9. What is Person 1's race? Mark  one or more boxes.

White

Black, African Am., or Negro

American Indian or Alaska Native — Print name of enrolled or principal tribe. ▾

Asian Indian     Japanese     Native Hawaiian

Chinese     Korean     Guamanian or Chamorro

Filipino     Vietnamese     Samoan

Other Asian — Print race, for example, Hmong, Laotian, Thai, Pakistani, Cambodian, and so on. ▾

Other Pacific Islander — Print race, for example, Fijian, Tongan, and so on. ▾

Some other race — Print race. ▾

Figure 1: The race question on the Decennial Census form in 2010.

I further note that Plan A, the first alternative plan presented in my report of December 10, has two majority-Black districts by any definition of Black that is plausibly used for VRA purposes: Any-Part-Black VAP, Black-Alone VAP, or Black Citizen VAP.<sup>2</sup>

	Black-Alone VAP	Any-Part-Black VAP	Black Citizen VAP
CD2	.5001	.5137	.5205
CD7	.5030	.5150	.5240

Table 1: Statistics for CD2 and CD7 in Plan A

### **Future inquiry via voter registration**

There is another source that could be useful to support the question of Black self-identification in Alabama: the voter registration file, in which citizens are asked to identify their race with a single choice. Counsel is currently attempting to secure a geocoded voter registration file. If I am provided with that resource in the near future, I hope to provide a supplemental report with the corresponding analysis.

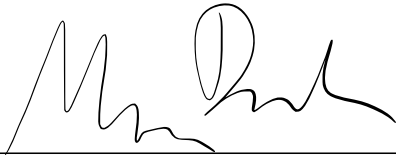
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<sup>2</sup>As explained in the supplemental material to my initial report, the BCVAP is estimated by using a special tabulation of the American Community Survey to calculate the citizenship rate for Black residents in the tract to which each block belongs, then applying that rate to the BVAP, in this case the Any-Part-Black VAP.



I declare under penalty of perjury that the foregoing is true and correct.

Executed this 20th day of December, 2021.



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Moon Duchin